Networks Research Lab (NetRL)

http://www.NetRL.cs.ucy.ac.cy
Large Scale Networks: Global stability results for arbitrary topologies

• address *largeness* - formally prove behaviour in a computer network of arbitrary topology

• extended work to stability issues for distributed CC schemes, which do not require maintenance of per flow states within network and achieve *max-min allocation*
  – some local stability results reported for various *max-min* congestion controllers proposed in literature
  – however, establishment of *global asymptotic stability* remained an open challenging research topic

• our recent work shows *global asymptotic stability for*
  – a *decentralized max-min congestion controller for networks of arbitrary topology*
  – for a simple integration strategy which guarantees that the user sending rates converge asymptotically to *max-min allocation* values for any arbitrary feasible initial condition
  – analytical results also confirmed with ns-2 discrete event simulations
Large Scale Networks: Global stability results for arbitrary topologies

Objective:

design operators $H(.)$, $F(.)$ $V(.)$ and $G(.)$ s.t. $r_i(t) \geq 0$, $\forall t \geq 0$, $\forall i \in I$ and

controller problem:

- design controller functions $H(.)$, $F(.)$, $V(.)$ and $G(.)$ such that
- sending rates converge asymptotically to unique set of values which achieve max-min fairness.
III. CONGESTION CONTROL ALGORITHM

We consider the following decentralized congestion control algorithm:

\[ r_i = h(q_i) = q_i, \quad \forall i \in I \]  

(14)

\[ q_i = F_i(p) = \min_j a_{ij}p_j, \quad \forall i \in I \]  

(15)

\[ \dot{z}_j = v(z_j, y_j) = Pr[C_j - y_j], \quad z_j(0) = z_j \geq 0, \quad \forall j \in J \]  

(16)

\[ p_j = g(z_j, y_j) = z_j \quad \forall j \in J \]  

(17)

where \( F_i(\cdot) \), \( i \in I \) denotes the \( i \)th element of the vector valued function \( F(\cdot) \) and \( \dot{z}_j = Pr[C_j - y_j] \) is defined as follows:

\[ \dot{z}_j = \begin{cases} 
C_j - y_j & \text{if } z_j < K \\
C_j - y_j & \text{if } z_j \geq K, C_j - y_j < 0 \\
0 & \text{otherwise}
\end{cases} 

(18)

Integrating excess capacity at each link

GLOBAL ASYMPTOTIC STABILITY

proven that system resulting from application of proposed CC scheme to a network of arbitrary topology guarantees feasible sending rates which converge to the unique set of values which achieve max-min fairness for any arbitrary feasible initial condition.

**Theorem 1:** The congestion control algorithm (14)-(17), when applied to system (8)-(12), guarantees that the controller states \( \{p_j, j \in J\} \) are bounded, \( r_i(t) \geq 0, \forall t \geq 0, \forall i \in I \) and

\[ \lim_{t \to \infty} r(t) = r^* \]  

(21)

for any feasible initial condition \( \{r_i(0) \geq 0, i \in I\} \), where \( r^* \) is the max-min vector of sending rates given by (13).

Ignore
- propagation delays
- queue dynamics